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COMPARISON OF NON-SPLIT AND SPLIT DELIVERY STRATEGIES FOR THE HETEROGENEOUS VEHICLE ROUTING PROBLEM

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ABSTRACT

This paper considers fresh goods distribution of a retail chain store in Izmir. The problem is formulated as a vehicle routing problem with a heterogeneous fleet. Although there are exact algorithms available in the literature, to the best of our knowledge, none of them is able to solve large scale instances optimally. The proposed algorithm decomposes the main problem into subproblems and simultaneously allocates vehicles to a number of NP-complete subproblems. Then integer programming is employed to solve subproblems. Also, non-split and split delivery strategies are tested for the distribution. Solutions of both strategies are compared with the current performance of the firm. Results indicated considerable improvement in the performance.

Key words: Heterogeneous Vehicle Routing Problem, Split Deliveries, Integer Programming.

HETEROJEN FİLOLU ARAÇ ROTALAMA PROBLEMİ İÇİN BÜTÜNLEŞİK VE BÖLÜNMÜŞ DAĞITIM STRATEJİLERİNİN KARŞILAŞTIRILMASI

ÖZET

Bu çalışma merkezi İzmir'de bulunan bir market zincirinin taze gıda dağıtımını incelemektedir. Problem, literatürde hiçbir yöntemin en iyi çözüm sağladığı ispat edilmemiş, heterojen filolu araç rotalama problemi olarak kurgulanmıştır. Önerilen çözüm algoritması ana problemi alt probleme ayrıştırıp, her alt probleme gerekli araçları atamaktadır. Daha sonra, alt problemler tamsayı programlama ile çözülmektedir. Aynı zamanda, birleşik teslimat ve ayrışık teslimat stratejileri bu yöntem içinde test edilmiştir. Sonuçlar firmanın şu anki dağıtım performansı ile karşılaştırılmıştır. Önerilen algoritma her iki strateji ile de mevcut performanstan daha iyi sonuçlar elde etmiştir.

Anahtar Sözcükler: Heterojen Filolu Araç Rotalama Problemi, Bölünmüş Dağıtım, Tamsayı Programlama.

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1. INTRODUCTION

Vehicle routing problem (VRP) has received a lot of attention in the Operations Research (OR) literature for its commercial value. VRP consists of designing *m* vehicle routes to minimize total cost, each starting and ending at the depot such that each customer is visited exactly once. As the number of demand nodes (customers) and vehicles increases, solution time increases non-polynomially. Thus many researchers have dedicated their researches to develop efficient algorithms for dealing with VRP and its extensions.

The classical VRP, which is called capacitated VRP (CVRP), can be simply stated as the problem of determining optimal routes through a set of locations and defined on a directed graph $G = (N, A)$ where $N = (n_0, n_1,\ldots, n_n)$ is a vertex set and $A= ((n_i, n_j) : n_i , n_j \in N, i\neq j)$ is an arc set. Vertex n_0 represents a depot node where a fleet $V =(v_1,\ldots, v_n)$ of vehicles exist with an identical and uniform capacity Q. All remaining vertices represent customers. A non-negative (distance/cost) matrix $C=(c_{ij})$ is defined on A. A non-negative weight d_i is associated with each vertex to represent the customer demand at n_i, and the total demand assigned to any route may not exceed the vehicle capacity Q. Thus, CVRP aims at determining vehicle routes of minimal total cost, each starting and ending at the depot, so that every customer is visited exactly once. A typical mathematical formulation for the single depot VRP is given in the following where X_{ijv} is a binary decision variable indicating whether vehicle v goes from n_i to n_j.

Formulation 1

$$Minimize \sum_i \sum_j \sum_v X_{ijv} * c_{ij} \qquad (1.a)$$

Subject to

$$\sum_j \sum_v X_{ijv} = 1 \qquad \forall i \in N \qquad (1.b)$$

$$\sum_i \sum_v X_{ijv} = 1 \qquad \forall j \in N \qquad (1.c)$$

$$\sum_i X_{i0v} \leq 1 \qquad \forall v \in V \qquad (1.d)$$

$$\sum_j X_{0jv} \leq 1 \qquad \forall v \in V \qquad (1.e)$$

$$\sum_i X_{ikv} = \sum_j X_{kjv} \qquad \forall k \in N, \forall v \in V \qquad (1.f)$$

$$\sum_i \left(\sum_j X_{ijv} \right) * d_i \leq Q \qquad \forall v \in V \qquad (1.g)$$

$$X_{ijv} \in Z \qquad \forall i, j \in N, \forall v \in V \qquad (1.h)$$

In the objective function of Formulation 1, the total distance traveled is minimized. By Constraints 1.b and 1.c, each node is visited exactly once. Constraint 1.d and 1.e state that every vehicle must go out of and into the depot node. Constraints 1.f assure that a vehicle ingoing to a node must leave that node. 1.g states that the capacities of vehicles should not be exceeded. Finally, 1.h eliminates subtours where,

$$Z = \left\{ (X_{ijv}) : \sum_{i \in B} \sum_{j \in B} X_{ijv} \leq |B| - 1, \forall B \subseteq V / \{0\}; |B| \geq 2 \right\}$$

In CVRP, all vehicles are assumed to be identical in capacity and cost. However, in real life problems, there exist a fixed fleet of vehicles mostly made up of different types with different capacities. Also these vehicles may have different fixed and variable traveling costs. When this is the case, the problem is formulated as a heterogeneous VRP (HVRP) for which due to its high computational complexity, no exact algorithm has ever been designed to solve yet. HVRP is studied under two different assumptions in literature. In the first one, it is assumed that there is an unlimited number of vehicles of each type. Hence the problem is to construct the optimum fleet. In the second one, it is assumed that there is a fixed fleet available and the problem is to make the optimum use of this fleet.

In this paper, the fresh goods distribution of a retail chain store in Turkey is handled. The problem is to perform the weekly distribution of fresh goods to the chain stores spread in the west and south regions of Turkey. It is formulated as a HVRP with a fixed fleet. A cluster first route second type of algorithms has been designed to solve the problem.

The need for this study has arisen from the necessity of an efficient distribution planning for the retail chain store. The proposed algorithm begins with the decomposition of HVRP into smaller scale ones and assigning vehicles to each subproblem. Then at second phase, each subproblem is solved by integer programming (IP).

The problem here, is not only to design vehicle routes but to determine a distribution strategy at the same time. Thus, in the second phase of the algorithm, the subproblems are first handled assuming that delivery of a customer cannot be split between vehicles. Then they are resolved considering split deliveries. Split Delivery VRP (SDVRP) is a relaxation of the capacitated VRP. In CVRP a customer can only be visited by one vehicle (as long as the demand does not exceed the capacity of the vehicle). On the other hand, in SDVRP the deliveries of a customer can be split between two or more vehicles. However, including this assumption does not make the problem easier to be solved. It is still NP-hard and to the best of our knowledge, no exact algorithm exists for SDVRP.

In the results of the study the solutions achieved under the two strategies; capacitated (non-split) and split delivery; are compared with the current operational performance of the firm. Computational results showed that both are dominant over the current performance of the firm.

The rest of the paper is organized as follows. In Section 2, a brief literature review on HVRP and SDVRP is given. In Section 3, the proposed algorithm is defined in detail. Section 4 gives the application of the proposed algorithm to the distribution problem of the retail chain store. Finally, conclusions are given in section 5.

2. LITERATURE REVIEW

HVRP is studied in two different versions in literature. Some of the researchers make an assumption that there is an unlimited number of vehicles of each type. They try to find the optimal set of vehicles to be scheduled in the problem. This is called the fleet size and mix VRP (FSMVRP). On the other hand, some researchers study the case where there is a fixed vehicle fleet. They try to schedule this fleet of vehicles to the customers in an optimal way. This problem is called heterogeneous fixed fleet VRP (HFVRP). Although HFVRP is more realistic than FSMVRP, it has attracted less attention in literature. Some of the earlier studies considering FSMVRP are Golden et al. (1984), Ulusoy (1985), Desrochers and Verhoog (1991). More recently, researchers have started applying more sophisticated approaches to FSMVRP. For instance, Salhi and Sari (1997) proposed a multi-level composite heuristic which simultaneously allocates customers to depots and determines the best fleet composition for the delivery routes. Ochi et al. (1998) used parallel GA together with scatter search to solve the problem. A tabu search heuristic is presented for FSMVRP by Gendreau et al. (1999). Liu and Shen (1999) presented a route construction method for FSMHVRP based on several different insertion heuristics. Lima et al. (2004) proposed a GA methodology. Taillard (1999) presented a heuristic column generation method. Choi and Tcha (2007) have also employed column generation. They have built an integer programming model and solved the linear relaxation by column generation technique.

As stated before, the other version of HVRP is where there is a heterogeneous fixed fleet of vehicles. There exist fewer studies in literature for HFVRP compared to FSMVRP. One of these studies belong to Taranatilis and Kironoudis (2001). They proposed an adaptive threshold accepting algorithm for HFVRP. In addition, Burchett and Campion (2002) applied tabu search to HFVRP in grocery supply industry. Faulin (2003) has also handled the logistics problem of a company in Spain and developed a MIXALG procedure for the case. Another study in this area is Moghaddom et al. (2006). The authors have proposed a linear integer programming model for the and solved the model using SA hybridized with nearest neighborhood heuristic. Also, Li et al. (2007) developed a record-to-record travel algorithm for the heterogeneous fleet.

Similar to HFVRP, there are very few studies concerning SDVRP in literature. One of these studies is Dror et al. (1994). The authors formulated the problem as an integer linear program. Then branch and bound algorithm is applied with the relaxation of constraints. However, this method turned out to be applicable only for small size problems. For larger problems, branch and bound is not able to work out the solution. Frizzel and Giffin (1995) developed three heuristics for SDVRP and tested these on some benchmark problems. SDVRP is formulated as a dynamic program (DP) with infinite number of states and solution spaces by Lee et al. (2002). Similar to Dror et al. (1994), dynamic programming approach in Lee et al. (2002) cannot find the solution for large size problems. Ho and Haugland (2004) considered SDVRP with time windows. They presented a tabu search algorithm to solve the problem and analyzed the performance of the approach on problems with 100 distribution points. Recently, Archetti et al. (In Press) have studied and identified the distribution environments in which allowing split deliveries are more beneficial. Moghaddam et al. (2007) also studied split deliveries and developed a simulated annealing approach.

In today's world, timeliness is very important to have a competitive edge in distribution. That means fast and effective decision making is as important as efficient distribution. Therefore some researchers have studied real-time vehicle routing (Du et al., 2006; Chen et al., 2006).

Similarly, the purpose of this study is to find an efficient solution to the distribution problem of the retail chain store in an effective time. IP applications have been successful in small size CVRPs. In larger scale problems, performance of IP have deteriorated dramatically. This fact has led this study to search a way to apply IP to larger scale HVRPs. Therefore, in this study a new algorithm is designed for HVRPs in which both split delivery and non-split delivery strategy is tested. The solutions are compared both in terms of cost and time.

3. THE PROPOSED ALGORITHM

The algorithm proposed is a cluster first route second type of algorithm. These algorithms are based on the idea of splitting a large size problem into subproblems and solving them faster. They have found

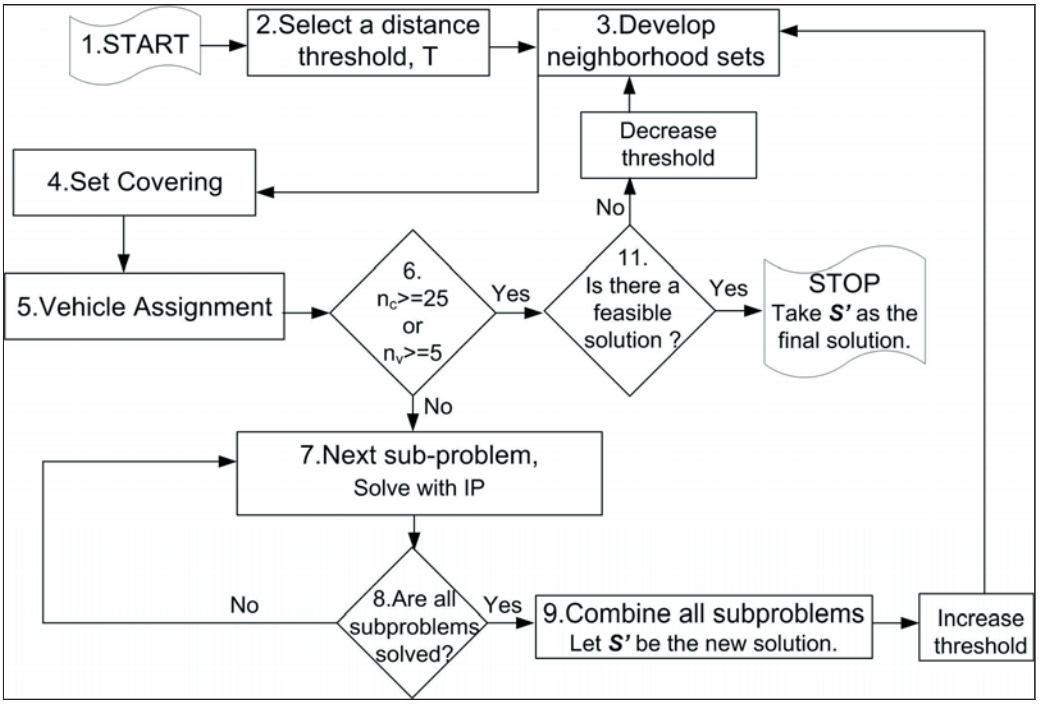

Figure 1. *Flow diagram of the algorithm*

wide application in literature. Laporte and Semet (2002) give some of the earlier studies in this area. One of the recent studies belongs to Dinçerler et al. (2004). The authors have used a cluster first route second algorithm to solve personnel transportation problem of a university in Ankara.

The success of these algorithms mainly depend on the how well the clusters are formed. Therefore, an effective procedure to split the problem into clusters followed by solving the clusters to optimality is expected to work well for HVRP.

In the clustering phase of the proposed approach, IP is used to solve a set covering problem. Then, from the sets selected, clusters are formed and vehicles are assigned through another IP model. In the finalization of the proposed approach, subproblems are solved first by IP. The flow of algorithm can be seen in Figure 1.

1. Start

2. Select a threshold *T*. Let there be "n_c" customers to be served.

3. Develop the neighborhood sets of each customer within distance *T* to the customer. This makes up totally "n_c" sets.

4. Set Covering: In this phase, among the "n_c" sets developed in step 3, a number of them are selected to cover all customers in the most centralized way. An integer programming model, given in Formulation 2, is used in this step. The notation used in Formulation 2 is given in Table 1.

Formulation 2

$$\text{Model SC}:$$

$$Min\ z_1 = \sum_{j \in Stores} \sum_{i \in \text{Stores}} \mathrm{M}_{ij} D_{ij} t_i \qquad (2.a)$$

subject to

$$\sum_{i \in Stores} \mathrm{M}_{ij} t_i >= 1 \qquad \forall j \in Stores \qquad (2.b)$$

$$t_i \in \{0,1\} \qquad \forall i \in Stores \qquad (2.c)$$

Objective function 2.a, does not only select arbitrarily the minimum number of sets but rather, it minimizes the distances between the central point and the neighborhood points. Constraint 2.b states that all demand nodes should be covered at least once and 2.c are the binary constraints.

5. Vehicle Assignment: The sets selected in step 4 may be intersecting. These should be turned into non-intersecting sets, and the necessary vehicles should be assigned to each set in order to develop subproblems. When assigning the vehicles their fixed costs should be taken into consideration. In order to do so, an integer programming model, given in Formulation 3, is built. Notation used in Formulation 3 is given in Table 2.

Table 1. Notation used in Model SC.

Parameters :

n_c : *Number of nodes to be served.*

D_{ij} : *Distance from node i to node j*, $i : 0..n_c,\ j : 0..n_c.$

$$M_{ij} : \begin{cases} 1 \text{ if set } i \text{ convers node } j. \\ 0 \text{ otherwise} \end{cases} \qquad , i : 1..n_c,\ j : 1..n_c.$$

Sets :

$Stores = \{1,2,\ldots\ldots,n_c\}$

Decision Variables :

$$t_i : \begin{cases} 1 \text{ if set } i \text{ is selected to be a subproblem} \\ 0 \text{ otherwise} \end{cases} \qquad , i{:}1..n_c$$

Table 2. Notation used in Model VA.

Parameters :
n_v : *Number of vehicles.*
$Demand_j$:*Demand of node* $j, j : 0..n_c$.
$Routecap_j$: *Total demand of route* j.
Cap_v : *Capacity of vehicle* $v, v : 1..n_v$.
$Cost_v$: *Fixed cost of vehicle* $v, v : 1..n_v$.
Sets :
$T^* = \{t_i^* : i \in Stores\}$: *Optimum solution of Model SC.*
$Subproblems = \{i \in Stores : t_i^* = 1\}$.
$Vehicles = \{1,2,.....,n_v\}$
Decision Variables :
Clustering Phase
$y_{ij} : \begin{cases} 1 \text{ if customer } j \text{ is assigned to subproblem } i \\ 0 \text{ if customer } j \text{ is not included in subproblem } i \end{cases} \quad , i \in Subproblems \;\; , j \in Stores.$
$w_{iv} : \begin{cases} 1 \text{ if vehicle } v \text{ is assigned to subproblem } i \\ 0 \text{ otherwise} \end{cases} \quad , i \in Subproblems \;\; , v{:}1..n_v.$

The objective of the model is to assign vehicles to each subproblem such that total fixed cost of vehicles is minimized (3.a). At the same time, the model assigns each customer to only one of the sets (found in step 4) in which it appears (3.b and 3.c). In addition, constraints 3.d and 3.e assure that the total capacity

Formulation 3

Model VA :

$$Min.\ z_2 = \sum_{i \in Subproblems} \sum_{v \in Vehicles} Cost_v w_{iv} \tag{3.a}$$

subject to

$$\sum_{i \in Subproblems} y_{ij} = 1 \qquad \forall j \in Stores \tag{3.b}$$

$$y_{ij} \le M_{ij} \qquad \forall i \in Subproblems, j \in Stores \tag{3.c}$$

$$Routecap_i = \sum_{j \in Stores} y_{ij} Demand_j \qquad \forall i \in Subproblems \tag{3.d}$$

$$\sum_{v \in Vehicles} Cap_v w_{iv} \ge Routecap_i \qquad \forall i \in Subproblems \tag{3.e}$$

$$\sum_{i \in Subproblems} w_{iv} \le 1 \qquad \forall v \in Vehicles \tag{3.f}$$

$$w_{iv} \in \{0,1\} \qquad \forall i \in Subproblems, \forall v \in Vehicles \tag{3.g}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in Subproblems, \forall j \in Store \tag{3.h}$$

of vehicles assigned to a subproblem must be greater than or equal to the total demand of customers in that subproblem. Constraint 3.f states that a vehicle can only be allocated to at most one subproblem. Finally, 3.g and 3.h are binary constraints. The output of the model determines the subproblems as well as the vehicles assigned to each one.

6. When the main problem is decomposed into subproblems, each one will be handled on its own. However, every time the threshold is increased, the subproblems get larger in size. Hence, the solution time increases quadratically. At a certain level of T, the optimal solution cannot be found. Therefore the algorithm should be stopped at that level of T. After a number of various experiments, that level is determined to be 25 nodes or 5 vehicles. When one of these limits are exceeded, then go to step 11.

7. At this step IP is employed to solve subproblems. Each $Subproblem_k$ is solved using Formulation 4. Notation used in Formulation 4 is given in Table 3.

Table 3. Notation used in Model IPM.

Parameters :
Noofsubtours: No of subtours identified by the OPL Script Algorithm (In IP approach of routing phase).
$subtours_{ti}$ *: The* i^{th} *node which is included in subtour* $t, t: 1..Noofsubtours$.
rhs_t*:Maximum allowable value to break subtour t,* t:1..*Noofsubtours.*
Sets :
$Y^* = \{y^*_{ij} : i \in Subproblems, j \in Stores\}$*:Optimum solution of Model VA.*
$Stores_k = \{j \in Stores : y^*_{kj} = 1\} \quad \forall k \in Subproblems; Let\ \lvert Stores_k \rvert = a_k \quad \forall k \in Subproblems$
$Total_k = Stores_k \cup \{0\}$*;"0" denotes depot node.*
$Subtourrange = \{1,2,..., Noofsubtours\}$
Decision Variables :
Routing Phase
$X_{ijv} : \begin{cases} 1 & \text{If vehicle v travels from node i to node j.} \\ 0 & \text{Otherwise.} \end{cases} \quad ,i : 0..n_c, j : 0..n_c, v : 1..n_v.$

Formulation 4

Model IPM :

$$Minimize \sum_{i \in Total} \sum_{j \in Total} \sum_{v \in Vehicles} X_{ijv} * D_{ij} \tag{4.a}$$

Subject to

$$\sum_{j \in Total_k} \sum_{v \in Vehicles} X_{ijv} = 1 \qquad \forall i \in Stores_k \tag{4.b}$$

$$\sum_{i \in Total_k} \sum_{v \in Vehicles} X_{ijv} = 1 \qquad \forall j \in Stores_k \tag{4.c}$$

$$\sum_{i \in Total_k} X_{i0v} = 1 \qquad \forall v \in Vehicles \tag{4.d}$$

$$\sum_{j \in Total_k} X_{0jv} = 1 \qquad \forall v \in Vehicles \tag{4.e}$$

$$\sum_{i \in Total_k} X_{ikv} = \sum_{j \in Total} X_{kjv} \qquad \forall k \in Total_k, \forall v \in Vehicles \tag{4.f}$$

$$\sum_{i \in Total_k} \left(\sum_{j \in Total_k, j \neq i} X_{ijv} \right) * Demand_i \leq Cap_v \qquad \forall v \in \ Vehicles \qquad (4.g)$$

$$\sum_{i \in [1..a_k]} X_{subtours_{ti} subtours_{ti+1} v} \leq rhs_t \qquad \forall t \in \ Subtourrange, \forall v \in \ Vehicles \qquad (4.h)$$

$$\sum_{i \in [1..a_k]} X_{subtours_{ti+1} subtours_{ti} v} \leq rhs_t \qquad \forall t \in \ Subtourrange, \forall v \in \ Vehicles \qquad (4.i)$$

$$X_{ijv} \in \{0,1\} \qquad \forall i, j \in \ Stores_k, \forall v \in \ Vehicles \qquad (4.j)$$

The objective function (4.a), minimizes total distance traveled. Constraints 4.b and 4.c assure that each customer is visited exactly once (non-split deliveries). In addition, each vehicle should visit the depot once since the vehicles assigned to the problem are known in advance (from Vehicle Assignment model, given in Formulation 2). This is included in the model with constraints 4.d and 4.e. Constraint set 4.f provides that a vehicle visiting a customer should leave that customer. It is stated by constraint set 4.g that capacity of vehicles should not be exceeded. If a single vehicle is assigned to the subproblem, then it becomes a traveling salesman problem. In this case constraint set 4.g becomes unnecessary. 4.j constraints are the binary constraints.

Constraints 4.h and 4.i are subtour elimination constraints. However, when all subtour constraints are included, the model becomes insolvably large. Therefore, an OPL Script algorithm which looks at the solution, finds the subtours and adds them into the IP model is integrated with it. The OPL Script (ILOG, 2003) algorithm for model IPM is given in the following:

1. Start;
2. Solve model IPM, let *S* be the solution;
3. If *S* does not contain any subtours then go to 7;
4. Identify subtours;
5. Add corresponding subtour elimination constraints to the IP Model;
6. Go to 2;
7. *S* is the optimum solution;

In other words *subtours* matrix contains a subtour in each of its rows. The matrix is empty at the very first iteration. That is, model IPM is solved without any subtour constraints at the beginning. Then OPL Script (ILOG, 2003) algorithm takes this solution, identifies the subtours and adds them to the *subtours* matrix. IPM is resolved with these subtour constraints (3.h and 3.i). The procedure goes on iteratively until no subtours exist in the solution, *S*. Then, *S* becomes the optimum.

The model and the script algorithm are written and solved in ILOG OPL Studio 3.7 (ILOG, 2003).

8. If all subproblems are finished then go to step 9, else go to step 7.

9. Combine the solutions of all subproblems to give the solution of the main problem.

10. Increase threshold by '*s*' units, T:=T+*s*. Go to 3.

11. If there is there a feasible solution achieved (from the previous iteration) then the algorithm STOPS. However, if no solution is achieved yet, then the procedure goes on by lowering the threshold and to Step 3.

4. APPLICATION OF THE PROPOSED ALGORITHM TO REAL LIFE CASE

In this study the distribution of fresh goods from a central depot to retail stores is handled. The depot belongs to a retail chain store located in Izmir, Turkey. There are 41 demand points to which the depot should make deliveries three times every week (Figure

Figure 2. *Demand points of the retail chain store*

Table 4: Data belonging to vehicles available

Vehicle	Capacity	Fixed Cost
1	11484	1148.4
2	11144	1114.4
3	14314	1431.4
4	10514	1051.4
5	9980	998
6	11824	1182.4
7	10514	1051.4
8	15724	1572.4
9	12000	1200

2). The distances between demand points are in kilometers and measured from main roads. In other words, since there exist physical road and land restrictions, some of the distance figures may not satisfy triangle inequality. The firm owns 9 vehicles assigned for this distribution. Data belonging to vehicles are given in Table 4.

4.1 VRP With Non-Split Deliveries

In the solution procedure, the first threshold level is taken to be 80 km (T=80). When set covering model is solved, 12 sets are selected. This means that at least 12 vehicles are required. In this case the solution is infeasible (since there exist 9 vehicles). Therefore threshold is increased to 100 km. and after that it is increased by 20 km at each iteration. After selecting the subproblems, vehicles are assigned to them by model VA (Formulation 3).

Once clusters are formed, IP approach with non-split delivery strategy is applied in the routing phase. Threshold levels, number of vehicles selected and total cost values of each iteration can be seen in Table 5.

The algorithm is stopped when T=260 since n_c exceeded 25 at this level of threshold. The most recent solution (solution at T=240) is selected. In the result, 4 vehicles are assigned (Vehicles 1, 2, 4 and 5) with a fixed cost of 4311 YTL. Total traveling cost turned out to be 3715 YTL which make up a total cost of 8026 YTL. In Figure 3, the threshold levels and the corresponding total cost values can be seen. Accord-

Table 5: Threshold levels, total cost values at each iteration

Threshold	No. of Subproblems	No. of Vehicles	Fixed Cost	Traveling Cost	Total Cost
80	13	Infeasible			
100	12				
120	10				
140	6	6	6544	4659	11203
160	6	6	6793	4853	11646
180	5	5	5645	4774	10419
200	4	4	5052	4369	9421
220	4	4	4769	4072	8841
240	4	4	4311	3715	8026

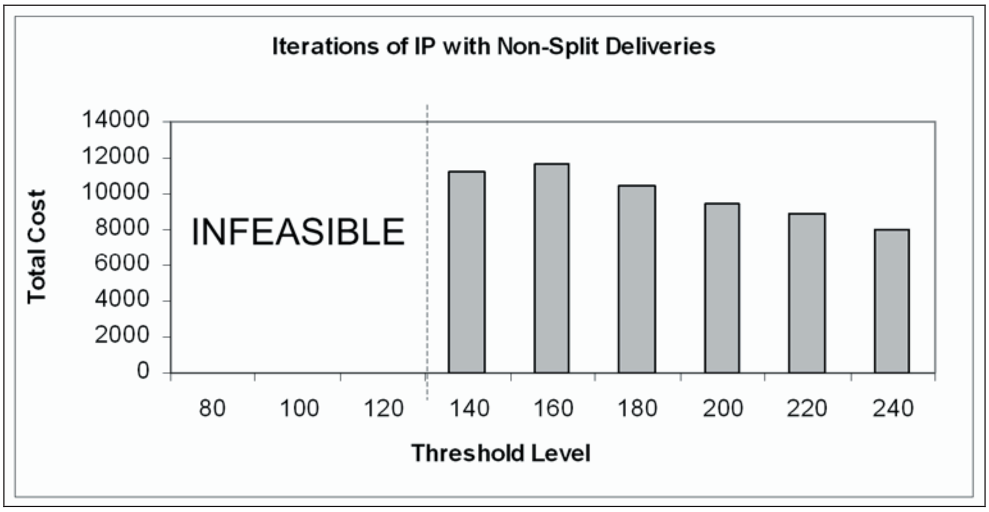

Figure 3. *Solutions of "non-split deliveries" according to the threshold level*

ing to the figure, it can be said that as threshold level increases, total cost decreases. Hence, the procedure approaches to the optimum solution of the global problem. The solutions can be seen in Table 6.

4.2 Employing Split Delivery Strategy

The problem is also solved considering split deliveries in order to determine a distribution strategy. Since the delivery of a customer can be split among two or more vehicles, another decision variable is necessary:

A_{iv}: Capacity of vehicle v allocated to customer i.

Split deliveries can be considered as a relaxation of the capacitated VRP. Therefore model IPM is revised according to split delivery assumptions. The objective function 4.a as well as the constraints 4.d, 4.e, 4.f, 4.h and 4.i and 4.j are reserved in the same form. However, constraints 4.b and 4.c; stating that all nodes should be visited exactly once; are turned into 4.b′ and 4.c′; stating that every node can be visited by one or more vehicles (split deliveries are allowed).

$$\sum_{j \in Total_k} X_{ijv} \le 1 \qquad \forall i \in Stores_k, \forall v \in Vehicles \qquad (4.b')$$

$$\sum_{i \in Total_k} X_{ijv} \le 1 \qquad \forall j \in Stores_k, \forall v \in Vehicles \qquad (4.c')$$

Constraints 4.g are completely removed. Capacity and demand requirements are stated using the decision variable A as:

$$\sum_{i \in Stores_k} A_{iv} \le Cap_v \qquad \forall v \in Vehicles \qquad (4.k')$$

$$\sum_{v \in Vehicles} A_{iv} = Demand_i \qquad \forall i \in Stores_k \qquad (4.l')$$

Constraints 4.k′ state that the goods carried in a vehicle should be less than or equal to its capacity and 4.l′ state that the goods carried in all vehicles for a node must be equal to its demand. Finally, the relationship between variables A and X are built by constraints 4.m′ and 4.n′ (B is a very large number) and non-negativity constraints are added by 4.p′.

$$A_{iv} \le B * \sum_{j \in Total_k} X_{ijv} \qquad \forall i \in Stores_k, \forall v \in Vehicles \qquad (4.m')$$

$$A_{iv} \ge \sum_{j \in Total_k} X_{ijv} \qquad \forall i \in Stores_k, \forall v \in Vehicles \qquad (4.n')$$

$$A_{iv} \ge 0 \qquad \forall i \in Stores_k, \forall v \in Vehicles \qquad (4.p')$$

The distribution problem is resolved considering split deliveries. The results of non-split delivery and split delivery strategies together with the current performance of the retail chain store can be seen in Table 6.

Table 6: Comparison of different distribution strategies and the current performance of the firm

	No. of Vehicles Allocated	Fixed Cost (YTL)	Traveling Cost (YTL)	Total Cost (YTL)	Solution Time (sec.)	Improvement (%)
Current Performance of the firm	5	5362	3908	9270	—	—
A - IP with non-split deliveries	4	4311	3715	8026	648*	13.42
B - IP with split deliveries	4	4311	3705	8016	657*	13.53

*: Solution times belong to routing phase of the algorithm (Model IPM). Solution times of the first two models are negligibly small.

As seen from the table, all strategies decrease the current distribution costs of the firm in a considerable way. The best solution cost is achieved by IP and by allowing split deliveries (Solution B). This strategy has led to 13.53% improvement in the current distribution performance. Also, the improvement achieved by split delivery strategy compared to non-split delivery is 0.1%. This was expected in the sense that splitting deliveries is kind of a relaxation of the problem.

5. CONCLUSION

In this paper, a two phase algorithm is developed for the distribution problem of a retail chain store in Turkey. The problem is formulated as a HVRP and solved both under non-split delivery and split delivery assumptions. The routes of the two strategies can be seen in Figure 4.

Both solutions provided decrease in the distribution costs of the retail chain store. When the two strategies are compared within themselves, it is seen that split delivery provides a slightly better solution than the non-split delivery strategy. In addition, there has not been a significant change in solution times. This is an expected result since split delivery strategy is a relaxation of capacitated VRP. But it is still NP-hard and due to the increase in the number of variables and constraints, solution times increase in a small amount.

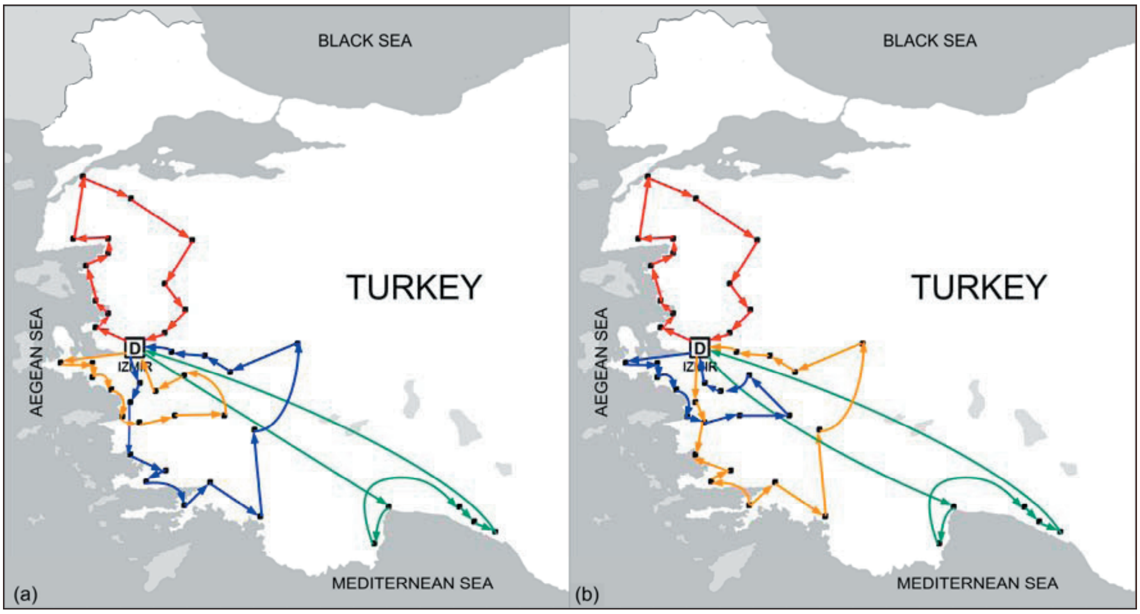

Figure 4. *Routes of (a) non-split delivery (b) split delivery distribution strategy*

In conclusion, allowing split deliveries is advised to the firm since it provides more improvement in solution cost in a reasonable time. However, for problems with larger number of customers or number of vehicles, solution times may go up drastically. In those cases, both strategies should be explored according to the firm's objectives.

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